

MAID ANALYSIS TECHNIQUES*

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MAID is a unitary isobar model for a partial wave analysis of pion photo- and electroproduction in the resonance region. It is fitted to the world data and can give predictions for multipoles, amplitudes, cross sections and polarization observables in the energy range from pion threshold up to $W = 2$ GeV and photon virtualities $Q^2 < 5$ GeV². Using more recent experimental results from Mainz, Bates, Bonn and JLab for Q^2 up to 4.0 GeV², the Q^2 dependence of the helicity couplings $A_{1/2}, A_{3/2}, S_{1/2}$ has been extracted for a series of four star resonances. We compare single- Q^2 analyses with a superglobal fit in a new parametrization of Maid2005. Besides the (pion) MAID, at Mainz we maintain a collection of online programs for partial wave analysis of η, η' and kaon photo- and electroproduction which are all based on similar footings with field theoretical background and baryon excitations in Breit-Wigner form.

1. Introduction

In 1998 the first version of MAID (MAID98) was developed and implemented on the web for an easy access for the whole community. MAID98 was based on a unitary isobar model constructed with a limited set of the seven most important nucleon resonances for pion photoproduction in Breit-Wigner form and a non-resonant background with Born terms and t-channel vector meson exchange contributions.¹ The model was unitarized for each partial wave up to the 2π threshold, the region where Watson's theorem is strictly valid. The model was later extended to eight resonances and the unitarization procedure was modified in accordance with the dynamical model (MAID2000). But yet the model was not fitted to the full world data base of pion photo- and electroproduction, and some background parameters were adjusted to multipoles from the SAID partial wave analysis. Results for finite Q^2 were mere predictions or extrapolations

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from photoproduction, where only the magnetic form factor of the Delta excitation was obtained from experimental results.

Since 2003 MAID has become a partial wave analysis program, where all parameters are fitted to experimental observables as cross sections and polarization asymmetries from pion photo- and electroproduction.

Besides the mostly developed Maid program for pion photo- and electroproduction on the nucleon, at Mainz we have developed and collected a series of programs for photo- and electroproduction of π, η, K and η' , covering the whole pseudoscalar meson nonet. The programs are available in the internet for online calculations under the URL <http://www.kph.uni-mainz.de/MAID>. In detail we can report on the following status:

- **MAID** is the main part of our project and describes the reaction $e + N \rightarrow e' + N + \pi$ in the kinematical range of $1.073 \text{ GeV} < W < 2 \text{ GeV}$ and $Q^2 < 5 \text{ GeV}^2$, see Refs.^{1,2}. The current version is *Maid2005*.
- **DMT** (Dubna-Mainz-Taipei) is a dynamical model based on a previous work of the Taipei group³. It also describes the reaction $e + N \rightarrow e' + N + \pi$ in the same kinematical range as Maid, see Ref.^{4,10}. The current version is *DMT2001*.
- **KAON-MAID** is an isobar model developed by Mart and Bennhold. It describes the reaction $e + N \rightarrow e' + \{\Lambda, \Sigma\} + K$ and is applicable in the kinematical range of $1.609 \text{ GeV} < W < 2.2 \text{ GeV}$ and $Q^2 < 2.2 \text{ GeV}^2$, see Ref.^{5,6}. The current version is *KaonMaid2000*.
- **ETA-MAID** is an isobar model for the reaction $e + N \rightarrow e' + N + \eta$. It exists in two versions, an older version (*EtaMaid2001*) that can give predictions for photo- and electroproduction of η from proton and neutron. It can be used in the kinematical range of $1.486 \text{ GeV} < W < 2.0 \text{ GeV}$ and $Q^2 < 5 \text{ GeV}^2$, see Ref.⁷.
The more recent version (*EtaMaid2003*) incorporates in addition an option to choose Regge tails for the t-channel vector meson exchange. It can only be applied to photoproduction on the proton in the kinematical range of $1.486 \text{ GeV} < W < 3.5 \text{ GeV}$ and $Q^2 < 5 \text{ GeV}^2$, see Ref.⁸.
- **ETA'-MAID** is an isobar model with t-channel ω, ρ Regge trajectories for the reaction $\gamma + p \rightarrow p + \eta'$ in the kinematical range of $1.896 \text{ GeV} < W < 3.5 \text{ GeV}$ and $Q^2 = 0$, see Ref.⁸. The current version is *EtaprimeMaid2003*.

- **DR-MAID** is a dispersion theoretical analysis of $e+N \rightarrow e'+N+\pi$ and is still in progress. It is based on fixed-t dispersion relations and uses as an input the imaginary parts of the MAID amplitudes, see Ref.⁹. The current version *DrMaid2004* is not yet available on the web.

2. The dynamical approach to meson electroproduction

In the dynamical approach to pion photo- and electroproduction^{3,10}, the t-matrix is expressed as

$$t_{\gamma\pi}(E) = v_{\gamma\pi} + v_{\gamma\pi} g_0(E) t_{\pi N}(E), \quad (1)$$

where $v_{\gamma\pi}$ is the transition potential operator for $\gamma^*N \rightarrow \pi N$, and $t_{\pi N}$ and g_0 denote the πN t-matrix and free propagator, respectively, with $E \equiv W$ the total energy in the CM frame. A multipole decomposition of Eq. (1) gives the physical amplitude⁴

$$t_{\gamma\pi}^{(\alpha)}(q_E, k; E + i\epsilon) = \exp(i\delta^{(\alpha)}) \cos \delta^{(\alpha)} \times [v_{\gamma\pi}^{(\alpha)}(q_E, k) + P \int_0^\infty dq' \frac{q'^2 R_{\pi N}^{(\alpha)}(q_E, q'; E) v_{\gamma\pi}^{(\alpha)}(q', k)}{E - E_{\pi N}(q')}] , \quad (2)$$

where $\delta^{(\alpha)}$ and $R_{\pi N}^{(\alpha)}$ are the πN scattering phase shift and reaction matrix in channel α , respectively; q_E is the pion on-shell momentum and $k = |\mathbf{k}|$ is the photon momentum. The multipole amplitude in Eq. (2) manifestly satisfies the Watson theorem and shows that the γ, π multipoles depend on the half-off-shell behavior of the πN interaction.

In a resonant channel the transition potential $v_{\gamma\pi}$ consists of two terms

$$v_{\gamma\pi}(E) = v_{\gamma\pi}^B + v_{\gamma\pi}^R(E), \quad (3)$$

where $v_{\gamma\pi}^B$ is the background transition potential and $v_{\gamma\pi}^R(E)$ corresponds to the contribution of the bare resonance excitation. The resulting t-matrix can be decomposed into two terms

$$t_{\gamma\pi}(E) = t_{\gamma\pi}^B(E) + t_{\gamma\pi}^R(E). \quad (4)$$

The background potential $v_{\gamma\pi}^{B,\alpha}(W, Q^2)$ is described by Born terms obtained with an energy dependent mixing of pseudovector-pseudoscalar πNN coupling and t-channel vector meson exchanges. The mixing parameters and coupling constants were determined from an analysis of non-resonant multipoles in the appropriate energy regions. In the new version

of MAID, the S , P , D and F waves of the background contributions are unitarized in accordance with the K-matrix approximation,

$$t_{\gamma\pi}^{B,\alpha}(\text{MAID}) = \exp(i\delta^{(\alpha)}) \cos\delta^{(\alpha)} v_{\gamma\pi}^{B,\alpha}(W, Q^2). \quad (5)$$

From Eqs. (2) and (5), one finds that the difference between the background terms of MAID and of the dynamical model is that off-shell rescattering contributions (principal value integral) are not included in MAID, therefore, after re-fitting the data, they are implicitly contained in the resonance part leading to dressed resonances.

Following Ref.¹, we assume a Breit-Wigner form for the resonance contribution $\mathcal{A}_\alpha^R(W, Q^2)$ to the total multipole amplitude,

$$\mathcal{A}_\alpha^R(W, Q^2) = \bar{\mathcal{A}}_\alpha^R(Q^2) \frac{f_{\gamma R}(W) \Gamma_R M_R f_{\pi R}(W)}{M_R^2 - W^2 - i M_R \Gamma_R} e^{i\phi}, \quad (6)$$

where $f_{\pi R}$ is the usual Breit-Wigner factor describing the decay of a resonance R with total width $\Gamma_R(W)$ and physical mass M_R . The expressions for $f_{\gamma R}$, $f_{\pi R}$ and Γ_R are given in Ref.¹. The phase $\phi(W)$ in Eq.(6) is introduced to adjust the phase of the total multipole to equal the corresponding πN phase shift $\delta^{(\alpha)}$. While in the original version of MAID only the 7 most important nucleon resonances were included with mostly only transverse e.m. couplings, in our new version all four star resonances below $W = 2$ GeV are included. These are $P_{33}(1232)$, $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$, $S_{31}(1620)$, $S_{11}(1650)$, $D_{15}(1675)$, $F_{15}(1680)$, $D_{33}(1700)$, $P_{13}(1720)$, $F_{35}(1905)$, $P_{31}(1910)$ and $F_{37}(1950)$.

The resonance couplings $\bar{\mathcal{A}}_\alpha^R(Q^2)$ are independent of the total energy and depend only on Q^2 . They can be taken as constants in a single- Q^2 analysis, e.g. in photoproduction, where $Q^2 = 0$ but also at any fixed Q^2 , where enough data with W and θ variation is available. Alternatively they can also be parametrized as functions of Q^2 in an ansatz like

$$\bar{\mathcal{A}}_\alpha(Q^2) = \bar{\mathcal{A}}_\alpha(0)(1 + \beta_1 Q^2 + \beta_2 Q^4 + \dots) e^{-\gamma Q^2}. \quad (7)$$

With such an ansatz it is possible to determine the parameters $\bar{\mathcal{A}}_\alpha(0)$ from a fit to the world database of photoproduction, while the parameters β_i and γ can be obtained from a combined fitting of all electroproduction data at different Q^2 . The latter procedure we call the ‘superglobal fit’. In MAID the photon couplings $\bar{\mathcal{A}}_\alpha$ are direct input parameters. They are directly related to the helicity couplings $A_{1/2}$, $A_{3/2}$ and $S_{1/2}$ of nucleon resonance excitation. For further details see Ref.¹¹.

3. Data analysis

The unitary isobar model MAID was used to analyze the world data of pion photo- and electroproduction. In a first step we have fitted the background parameters of MAID and the transverse normalization constants $\bar{\mathcal{A}}_\alpha^R(0)$ for the nucleon resonance excitation. The latter ones give rise to the helicity couplings shown in Tables 1 and 2. Most of the couplings are in good agreement with PDG and the GW/SAID analysis. It is very typical for such a global analysis, where about 15000 data points are fitted to a small set of 10-20 parameters, that the fit errors appear unrealistically small. Such errors only reflect the statistical uncertainty of the experimental errors, but the model uncertainty can be ten times larger. Therefore we do not report these fit errors which are very similar as in the GW02 fits. The only reliable error estimate can be obtained by comparing different analyses like SAID, MAID and coupled channels analyses.

In Fig. 1 we give a comparison between MAID and SAID for three important multipoles, $E_{0+}(S_{11})$, $M_{1-}(P_{11})$ and $E_{2-}(D_{13})$. For both analyses we show the global (energy dependent) curves together with the local (single energy) fits, where only data in energy bins of 10-20 MeV are fitted. Such a comparison demonstrates the fluctuations due to a limited data base, especially in the case of the Roper multipole M_{1-} . It also shows systematic differences between the MAID and SAID analyses in the real parts of E_{0+} and E_{2-} . Because of correlations between these amplitudes, these differences cannot be resolved with our current data base. Because of isospin 1/2, they can, however, lead to sizeable differences in the γ, π^+ channel, where the data base is still quite limited.

In a second step we have fitted recent differential cross section data on $p(e, e'p)\pi^0$ from Mainz¹⁴, Bates¹⁵, Bonn¹⁶ and JLab^{17,18,19}. These data cover a Q^2 range from $0.1 \cdots 4.0$ GeV² and an energy range 1.1 GeV $< W < 2.0$ GeV. In a first attempt we have fitted each data set at a constant Q^2 value separately. This is similar to a partial wave analysis of pion photoproduction and only requires additional longitudinal couplings for all the resonances. The Q^2 evolution of the background is described with nucleon Sachs form factors in the case of the s - and u - channel nucleon pole terms. At the e.m. vertices of the π pole and seagull terms we apply the pion and axial form factors, respectively, while a standard dipole form factor is used for the vector meson exchange. Furthermore, as mentioned above, we have introduced a Q^2 evolution of the transition form factors of the nucleon to N^* and Δ resonances and have parameterized each of

Table 1. Proton helicity amplitudes at $Q^2 = 0$ of the major nucleon resonances. The results from our own analyses with Maid2003 and the current Maid2005 version are compared to the Particle Data Tables¹² and the GW/SAID¹³ analysis. Numbers are given in units of $10^{-3} \text{ GeV}^{-1/2}$.

		PDG	GW02	MD03	MD05
$P_{33}(1232)$	$A_{1/2}$	-135 ± 6	-129 ± 1	-140	-137
	$A_{3/2}$	-255 ± 8	-243 ± 1	-265	-260
$P_{11}(1440)$	$A_{1/2}$	-65 ± 4	-67 ± 4	-77	-61
$D_{13}(1520)$	$A_{1/2}$	-24 ± 9	-24 ± 4	-30	-27
	$A_{3/2}$	166 ± 5	135 ± 4	166	161
$S_{11}(1535)$	$A_{1/2}$	90 ± 30	30 ± 4	73	66
$S_{31}(1620)$	$A_{1/2}$	27 ± 11	-13 ± 4	71	66
$S_{11}(1650)$	$A_{1/2}$	53 ± 16	74 ± 4	32	33
$D_{15}(1675)$	$A_{1/2}$	19 ± 8	33 ± 4	23	15
	$A_{3/2}$	15 ± 9	9 ± 4	24	22
$F_{15}(1680)$	$A_{1/2}$	-15 ± 6	-13 ± 4	-25	-25
	$A_{3/2}$	133 ± 12	129 ± 4	134	134
$P_{13}(1720)$	$A_{1/2}$	18 ± 30		55	73
	$A_{3/2}$	-19 ± 20		-32	-11

Table 2. Neutron helicity amplitudes at $Q^2 = 0$ of the major nucleon resonances. Notation as in Table 1.

		PDG	GW02	MD03	MD05
$P_{11}(1440)$	$A_{1/2}$	40 ± 10	47 ± 5	52	54
$D_{13}(1520)$	$A_{1/2}$	-59 ± 9	-67 ± 4	-85	-77
	$A_{3/2}$	-139 ± 11	-112 ± 3	-148	-154
$S_{11}(1535)$	$A_{1/2}$	-46 ± 27	-16 ± 5	-42	-51
$S_{11}(1650)$	$A_{1/2}$	-15 ± 21	-28 ± 4	27	9
$D_{15}(1675)$	$A_{1/2}$	-43 ± 12	-50 ± 4	-61	-62
	$A_{3/2}$	-58 ± 13	-71 ± 5	-74	-84
$F_{15}(1680)$	$A_{1/2}$	29 ± 10	29 ± 6	25	28
	$A_{3/2}$	-33 ± 9	-58 ± 9	-35	-38
$P_{13}(1720)$	$A_{1/2}$	1 ± 15		17	-3
	$A_{3/2}$	-29 ± 61		-75	-31

the transverse $A_{1/2}$ and $A_{3/2}$ and longitudinal $S_{1/2}$ helicity couplings. In a combined fit with all electroproduction data from the world data base of GWU/SAID¹³ and the data of our single- Q^2 fit we obtained a Q^2 dependent solution (superglobal fit).

In Fig. 2 we show our results for the $\Delta(1232)$, the $D_{13}(1520)$ and the $F_{15}(1680)$ resonances. Our superglobal fit agrees very well with our single- Q^2 fits, except in the case of the Δ resonance for the 2 lowest points of $S_{1/2}$ from our analysis of the Hall B data. Whether this is an indication

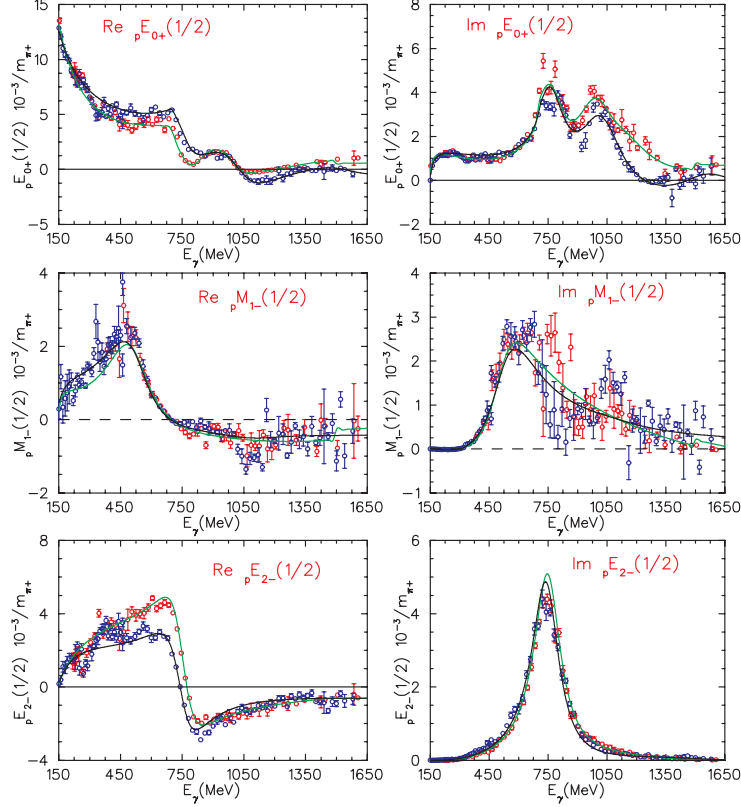


Figure 1. Comparison of MAID and SAID multipoles. The blue lines and points show the Maid2005 global and single-energy solutions, respectively. The global and s.e. SAID solutions are shown in green and red colours.

for a different Q^2 dependence has still to be investigated. Generally, all our single- Q^2 points are shown with statistical errors from χ^2 minimization only. A much bigger error has to be considered for model dependence.

In Fig. 3 we show our results for the helicity amplitudes of the Roper resonance $P_{11}(1440)$ and the $S_{11}(1535)$ in the region of $Q^2 < 1 \text{ GeV}^2$. In addition to our own single- Q^2 analysis we also compare to the analysis of Aznauryan and Burkert²⁰ who used both an isobar model similar to Maid and an analysis based on fixed- t dispersion relation. In general we get a good agreement with the results of Ref.²⁰. Only for the longitudinal excitation of the S_{11} resonance one may observe a different tendency of the Q^2 dependence, however, in this case the statistical fluctuations of our

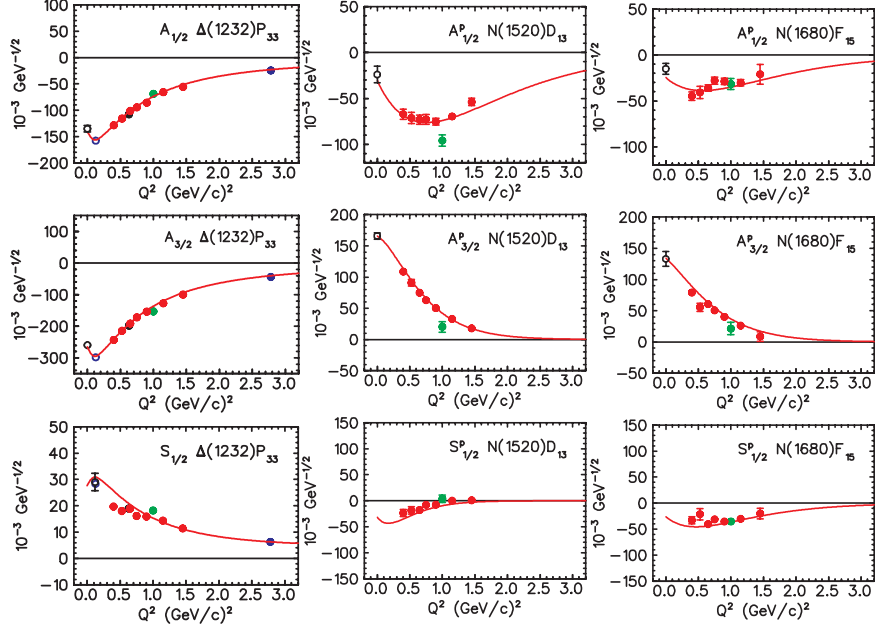


Figure 2. The Q^2 dependence of the transverse ($A_{1/2}, A_{3/2}$) and longitudinal ($S_{1/2}$) helicity couplings for the $P_{33}(1232)$, $D_{13}(1520)$ and $F_{15}(1680)$ resonance excitation. The solid curves show our superglobal fit. The data points at finite Q^2 are obtained from our single- Q^2 analysis using the data from MAMI and Bates for $Q^2 = 0.1 \text{ GeV}^2$, from ELSA for 0.6 GeV^2 , JLAB(Hall A) for 1.0 GeV^2 , JLab(Hall C) for 2.8 GeV^2 and JLab(Hall B) for the remaining points. At the photon point ($Q^2 = 0$) we show our result from Table 1 obtained from the world data base.

analysis is quite large.

Furthermore we also have some empirical results for the partial waves that are not shown here, but most of them come out of the fit with rather large errors bars in the single- Q^2 analysis. This gives us less confidence also for our superglobal fit. The reason for it is mainly that we have fewer data points to analyze at higher energies.

4. Conclusions

Using the world data base of pion photo- and electroproduction and recent data from Mainz, Bonn, Bates and JLab we have made a first attempt to extract all longitudinal and transverse helicity amplitudes of nucleon resonance excitation for four star resonances below $W = 2 \text{ GeV}$. For this purpose we have extended our unitary isobar model MAID

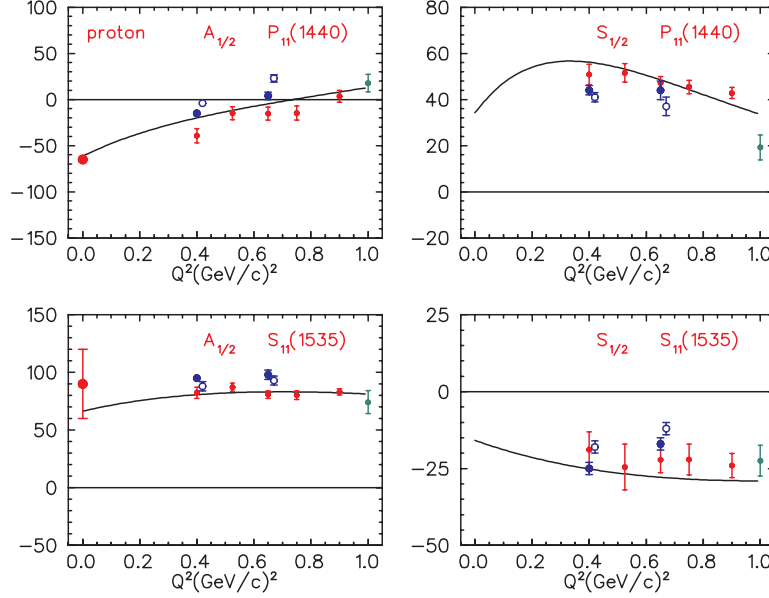


Figure 3. The Q^2 dependence of the transverse and longitudinal helicity amplitudes for the $P_{11}(1440)$ and the $S_{11}(1535)$ resonance excitation of the proton. The solid lines are the superglobal Maid2005 solutions. The solid red (gray) points are our single- Q^2 fits to the exp. data from CLAS/JLab¹⁸, the solid and open blue circles show the isobar and dispersion analysis of Aznauryan²⁰ using a similar data set.

and have parameterized the Q^2 dependence of the transition amplitudes. Comparisons between single- Q^2 fits and a Q^2 dependent superglobal fit give us confidence in the determination of the helicity couplings of the $P_{33}(1232)$, $P_{11}(1440)$, $S_{11}(1535)$, $D_{13}(1520)$ and the $F_{15}(1680)$ resonances, even though the model uncertainty of these amplitudes can be as large as 50% for the longitudinal amplitudes of the D_{13} and F_{15} .

For other resonances the situation is more uncertain. However, this only reflects the fact that precise data in a large kinematical range are absolutely necessary. In some cases double polarization experiments are very helpful as has already been shown in pion photoproduction. Furthermore, without charged pion electroproduction, some ambiguities between partial waves that differ only in isospin as S_{11} and S_{31} cannot be resolved without additional assumptions. While all electroproduction results discussed here are only for the proton target, we have also started an analysis for the neutron, where much less data are available from the world data base and no new data has been analyzed in recent years. Since we can very well rely on

isospin symmetry, only the electromagnetic couplings of the neutron resonances with isospin $1/2$ have to be determined. We have obtained a global solution for the neutron which is implemented in MAID2005. However, for most resonances this is still highly uncertain. So it will be a challenge for the experiment to investigate also the neutron resonances in the near future.

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